Functions Day 4: In Class

Modeling with Exponential Equations

1. An alpaca ranch had a population of 187 alpacas on January 1, 2010. The farmers think that the population is growing at a rate of 5 alpacas per year.

a. Write an equation that represents the population of alpacas, A(*t*), *t* years after January 2010.

$A(t) = 187 + 5t$

b. What if the initial alpaca population had been something other than 187? Write a formula for the population of alpacas if the initial population had been any number *b*.

$A(t) = b + 5t$

c. Now, rewrite the formula from part (1b) but ~~consider~~ the alpacas were growing at a rate of *m* per year.

$A(t) = mt + b$

2. The ranchers realized their initial assumptions were not accurate, and now think that the alpaca population is increasing exponentially with a growth rate of 15%. Complete the following table:

$1 + 0.15$

Number of years since January 2010	Number of alpacas
0	187
1 $187 \cdot (1.15)$	215
2 $187(1.15)(1.15)$	247
3 $187(1.15)(1.15)(1.15)$	284
4 1	

a. Write an equation that represents the population of alpacas, A(*t*), *t* years after January 2010.

$A(t) = 187(1.15)^t$

b. What if the initial alpaca population had been something other than 187? Write a formula for the population of alpacas if the initial population had been any number *A*.

$A(t) = a(1.15)^t$

c. Now, rewrite the formula from part (2b) but consider if we had any growth factor *b*.

$A(t) = ab^t$

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3. For your 16th birthday, your grandparents gave you $120. You're considering investing it in a bank that has a 3% annual interest rate.

a. How much would you have in the bank after 1 year?

$m(t) = 120(1.03)^t$ $m(1) = 120(1.03)^1 = \$123.60$

b. How much would you have in the bank after 2 years?

$m(2) = 120(1.03)^2 =$

c. What is the annual growth rate? The annual growth factor?

3%   1.03

d. Write an equation that expresses how much you would have in the bank after *t* years.

$m(t) = 120(1.03)^t$

e. What if your grandparents had given you more, or less? Write an equation for any initial investment, *P*.

$m(t) = P(1.03)^t$

f. Suppose you found a bank with a different annual interest rate. Write an equation for any interest rate, *r*.

$m(t) = P(1+r)^t$

g. Suppose that instead of giving you interest every year, the bank gives you interest of 0.3% every month. Write an equation for the amount of money you will have after *t* years.

$M(t) = P(1+r)^{nt}$

t years
r years

$m(t) = P\left(1+\frac{r}{n}\right)^{nt}$

h. What would the annual interest rate be that would give me the same amount of money after the first year as doing the monthly plan in part g?

$0.3\% = 0.003$

$0.003 \cdot 12 \text{ month} = 0.036 = 3.6\%$

I. Which is better for you, 3% compounded annually, or 0.3% compounded monthly? Why?

Annual
$m(1) = 120(1+0.03)^1$
$= \$123.60$
$m(30) = 120(1.03)^{30}$
$= \$291.27$

monthly
$m(1) = 120(1+0.003)^{12}$
$= \$124.39$
$m(30) = 120(1.003)^{360}$ (12·30)
$= 352.79$

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Exponential Generic Formula is $f(x) = a \cdot b^x$.

Where *a* is the start, Initial, y-int, principal, and *b* is the factor of change.

Equivalently, when doing annually compounding interest with annual rate r, we use the equation $f(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$.

Where P is the principal and t is measured in years.

When given a interest rate for non annual interest r that compounds n times a year, the equation is given by $f(t) = P(1+r)^{n \cdot t}$.

Where P is the same as before and t is measured in monthly, semi annually (2), Daily, ...
n is how many times it divides in one year.

4. Dylan is opening a savings account with $500. Bank A offers a savings account with 0.15% interest compounded **monthly**. Bank B does not give interest, but instead gives a bonus of $10 per year.

a. Fill in the following table describing the difference between the banks.

	BANK A Exp	BANK B linear
Annual Growth Rate	0.15% / year	$10 / year
Annual Growth Factor	0.15% = 0.0015 $(1.0015)^{12}$	
Equation for the total money after t years	$M_E(t) = 500(1.0015)^{t \cdot 12}$	$M_A(t) = 500 + 10t$

b. Which bank is better to put the money in if Dylan is only planning on leaving it in for 6 years?

$M_E(6) = 500(1.0015)^{6 \cdot 12}$
$= \$556.98$

$M_A(t) = 500 + 10(6)$
$= 560$

c. What about if Dylan wants to leave it in for 15 years?