

## Properties of Rational Exponents

- 1) We have learned the properties with integer exponents. Sometimes, our exponents are \_\_\_\_\_ meaning they are \_\_\_\_\_.

Before we work with rational exponents, we need to practice operations with fractions.

2)  $\frac{2}{3} + \frac{5}{9}$

3)  $\frac{1}{4} + \frac{3}{5}$

4)  $\frac{2}{5} - \frac{1}{6}$

5)  $\frac{11}{12} - \frac{1}{3}$

6)  $\frac{2}{3} \cdot \frac{2}{5}$

7)  $\frac{1}{6} \cdot \frac{3}{7}$

8)  $\frac{2}{3} \div \frac{1}{2}$

9)  $\frac{4}{7} \div 14$

- 10) Now apply the same properties as above, but with rational exponents (fraction exponents)

*Rules stay the same!*

**Simplify.**

11)  $2k^{\frac{5}{3}} \cdot 3k^2$

*Handwritten work:*  
 $2 \cdot 3 \cdot k^{\frac{5}{3}} k^{\frac{2 \cdot 3}{3}}$   
 $6 \cdot k^{\frac{5}{3}} k^{\frac{6}{3}}$   
 $6k^{\frac{11}{3}}$

12)  $3a^{\frac{3}{2}} \cdot a$

*Handwritten work:*  
 $3a^{\frac{3}{2}} a^{\frac{2 \cdot 1}{2}}$   
 $3a^{\frac{5}{2}}$

13)  $2r^2 \cdot 3rr^{\frac{5}{3}}$

*Handwritten work:*  
 $2 \cdot 3 \cdot r^2 \cdot r^1 \cdot r^{\frac{5}{3}}$   
 $6 \cdot r^{\frac{6}{3}} r^{\frac{3}{3}} r^{\frac{5}{3}}$   
 $6r^{\frac{14}{3}}$

14)  $2k \cdot 3k^2$

*Handwritten work:*  
 $6k^3$

Simplify. Your answer should contain only positive exponents.

$$15) \frac{4n^{\frac{3}{2}}}{n^{\frac{3}{4}}} = 4 n^{\frac{3}{2}} n^{-\frac{3}{4}}$$

$$= 4 n^{\frac{3}{4}}$$

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$$16) \frac{3a^2}{3a^{\frac{3}{2}}}$$

$$\frac{N}{N} = 1$$

$$\frac{X^5}{X^7} = \frac{1}{X^2}$$

$$17) \frac{2m^{\frac{5}{3}}}{3m}$$

$$\left(\frac{2}{3}\right) \cdot \frac{m^{\frac{5}{3}}}{m^1}$$

$$18) \frac{r^{\frac{7}{4}}}{4r^2} = r^{\frac{7}{4}} \cdot 4^{-1} r^{-2}$$

$$\frac{1}{4} \cdot \frac{r^{\frac{7}{4}}}{r^2}$$

$$= 4^{-1} \cdot r^{\frac{7}{4}} r^{-\frac{8}{4}}$$

$$= \frac{4^{-1} \cdot r^{-\frac{1}{4}}}{1} = \frac{1}{4r^{\frac{1}{4}}}$$

$$19) \frac{x^2 x^{\frac{3}{2}}}{(x^{\frac{2}{3}})^{-\frac{1}{3}}} = \frac{x^2 x^{\frac{3}{2}}}{x^{\frac{2}{3}}}$$

$$= x^{\frac{2}{1}} x^{\frac{3 \cdot 3}{2 \cdot 3}} x^{-\frac{2 \cdot 2}{3 \cdot 2}}$$

$$= x^{\frac{2}{1}} x^{\frac{9}{2}} x^{-\frac{4}{6}} = \left(x^{\frac{17}{6}}\right)$$

$$20) \frac{\left(\frac{2}{r^3}\right)^{-1} \cdot r^{\frac{4}{3}}}{r^{\frac{3}{4}}} = \frac{r^{\frac{3}{1}} r^{\frac{4}{3}}}{r^{\frac{3}{4}}}$$

$$= \frac{r^{\frac{10}{3}}}{r^{\frac{3}{4}}} = r^{-\frac{1}{12}} = \frac{1}{r^{\frac{1}{12}}}$$

Let's review what the parts of a radical expression are:

21) 
$$\text{Base} \rightarrow X \quad \overset{2 \leftarrow \text{Power}}{\underset{3 \leftarrow \text{Root}}{\sqrt{\quad}}} \left( \underset{\text{Base}}{X} \right) \overset{2 \leftarrow \text{Power } (\wedge)}{\quad} \text{radicand}$$

22) What is  $x^{\frac{1}{2}} x^{\frac{1}{2}}$ ?  $= x^{\frac{1}{2} + \frac{1}{2}} = x = (x^{\frac{1}{2}})^2$

What about  $\underline{\quad} * \underline{\quad} = 25$ ? If the blanks are the same thing, what are they?

Therefore,  $x^{\frac{1}{2}} = ?$   $\sqrt{x}$

What is  $r^{\frac{1}{3}} r^{\frac{1}{3}} r^{\frac{1}{3}}$ ?

What about  $\underline{\quad} * \underline{\quad} * \underline{\quad} = 8$ ? If the blanks are the same thing, what are they?

Therefore,  $x^{\frac{1}{3}} = ?$

So when converting a rational exponent to a radical:

The numerator is the power of the radicand.

The denominator is the index of the root.

In other words,  $\sqrt{3} = \sqrt{3^1}$  can be written as  $3^{\frac{1}{2}}$ .

Example:  $4^{\frac{2}{3}} = \sqrt[3]{4^2} = (\sqrt[3]{4})^2$

Write each expression in radical form.

23)  $n^{\frac{5}{2}} = \sqrt[2]{n^5} = (\sqrt{n})^5$

24)  $n^{\frac{5}{4}} = \sqrt[4]{n^5}$

25)  $(5n)^{\frac{4}{3}} = \sqrt[3]{(5n)^4} = (\sqrt[3]{5n})^4$

26)  $(5m)^{\frac{3}{2}} = \sqrt{(5m)^3} = (\sqrt{5m})^3$

27)  $(2x)^{\frac{5}{6}} = \sqrt[6]{(2x)^5} = (\sqrt[6]{2x})^5$

28)  $r^{\frac{1}{3}} = \sqrt[3]{r}$

Write each expression in exponential form.

29)  $\sqrt[5]{p} = p^{\frac{1}{5}}$

30)  $(\sqrt{7r})^3 = (7r)^{\frac{3}{2}}$

31)  $\sqrt{2m} = (2m)^{\frac{1}{2}}$

32)  $\sqrt[3]{7n} = (7n)^{\frac{1}{3}}$

33)  $\sqrt[3]{3v} = (3v)^{\frac{1}{3}}$

34)  $(\sqrt{6x})^5 = (6x)^{\frac{5}{2}}$