

Exploring Laws of Exponents

Name Key.

What does an exponent tell you about the number?

Product of Powers Property

Complete the table below.

Expression	E·X·P·A·N·D·E·D Form	Exponential Form
$3^2 \cdot 3^5$	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	3^7
$2 \cdot 2^6$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^7
$b^5 \cdot b^8$	$b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$	b^{13}

1. Look at the table above. Compare the first and third column. Describe, using words, the relationship that you see between them.

2. Use your observations from above to fill in a generalized form:

$$a^m \cdot a^n = a^{m+n}$$

3. The rule you discovered is called the "product of powers." Use it to simplify the expressions below.

a. $2^6 \cdot 2^8$
 2^{14}

b. $(-7)^3 \cdot (-7)^1 \cdot (-7)^5$
 $(-7)^9$

c. $m^7 \cdot m^4 \cdot m^6$
 m^{17}

Power of a Power Property

Complete the table below.

Expression	E·X·P·A·N·D·E·D Form	Exponential Form
$(5^2)^3$	$5^2 \cdot 5^2 \cdot 5^2$	5^6
$(8^5)^2$	$8^5 \cdot 8^5$	8^{10}
$(x^3)^4$	$x^3 \cdot x^3 \cdot x^3 \cdot x^3$	x^{12}

4. Look at the table above. Compare the first and third column. Describe, using words, the relationship that you see between them.

5. Use your observations from above to fill in a generalized form:

$$(a^m)^n = a^{m \cdot n}$$

6. The rule you discovered is called the "power of a power." Use it to simplify the expressions below.

a. $(7^4)^9$
 $7^{4 \cdot 9}$
 7^{36}

b. $(k^{17})^2$
 $k^{17 \cdot 2}$
 k^{34}

c. $(w^{100})^5$
 $w^{100 \cdot 5}$
 w^{500}

Quotient of Powers Property

Complete the table below.

Expression	E·X·P·A·N·D·E·D Form	Exponential Form
$\frac{6^9}{6^4}$	$\frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6 \cdot 6}$	6^5
$\frac{100^5}{100^2}$	$\frac{100 \cdot 100 \cdot 100 \cdot 100 \cdot 100}{100 \cdot 100}$	100^3
$\frac{t^{15}}{t^8}$	$\frac{t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t}{t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t}$	t^7

7. Look at the table above. Compare the first and third column. Describe, using words, the relationship that you see between them.

8. Use your observations from above to fill in a generalized form:

$$\frac{a^m}{a^n} = a^{m-n}$$

9. The rule you discovered is called the "quotient of powers." Use it to simplify the expressions below.

a. $\frac{16^7}{16^3}$
 16^{7-3}
 16^4

b. $\frac{w^{25}}{w^{10}}$
 w^{25-10}
 w^{15}

c. $\frac{4^3 \cdot 4^7}{4^5} = \frac{4^{10}}{4^5}$
 $= 4^{10-5}$
 $= 4^5$

Zero Powers Property

Complete the table below.

Expression	E·X·P·A·N·D·E·D Form	Answer
5^4	$5 \cdot 5 \cdot 5 \cdot 5$	625
5^3	$5 \cdot 5 \cdot 5$	$= 125$
5^2	$5 \cdot 5$	$= 25$
5^1	5	$= 5$
5^0		$= 1$

10. Look at the table above. What is the pattern in the third column? Describe, using words, how that pattern helps you find 5^0 ?

Expression	E·X·P·A·N·D·E·D Form	Answer
3^4	$3 \cdot 3 \cdot 3 \cdot 3$	81 $\div 3$
3^3	$3 \cdot 3 \cdot 3$	27 $\div 3$
3^2	$3 \cdot 3$	9 $\div 3$
3^1	3	3 $\div 3$
3^0		1

11. Use your observations from above to fill in a generalized form:

$$a^0 = 1$$

12. The rule you discovered is called the "zero power property." Use it to simplify the expressions below.

a. $(x \cdot x^2 \cdot x^9)^0$
1

b. $\left(\frac{3^4}{3^0}\right)^2$
 $\left(\frac{3^4}{1}\right)^2 = (3^4)^2 = 3^8$

c. $(128,917,654^5)^0$
1

Negative Powers Property

Complete the table below.

Expression	E·X·P·A·N·D·E·D Form	Answer
4^2	4·4	16 $\div 4$
4^1	4	4 $\div 4$
4^0	1	1 $\div 4$
4^{-1}	$\frac{1}{4}$	$\frac{1}{4}$ $\div 4$
4^{-2}	$\frac{1}{4 \cdot 4}$	$\frac{1}{16}$ $\div 4$

13. In order to work up in exponents, you are increasing the number of times that you are multiplying the number (or variable).
If you are decreasing the exponent (the exponent is getting more negative) then you are dividing by the number (or variable).

14. Use your observations from above to fill in a generalized form:

$$\frac{a^{-m}}{1} = \frac{1}{a^m} \qquad \frac{1}{a^{-m}} = \frac{a^m}{1} = a^m$$

15. The rule you discovered is called the "negative power property." Use it to simplify the expressions below.

a. $\frac{y^{-5}}{1}$
 $\frac{1}{y^5}$

b. $\frac{(x^2)^{-5}}{1}$
 $\frac{1}{(x^2)^5}$
 $\frac{1}{x^{10}}$

c. $\left(\frac{z^0}{z^2 \cdot z^4}\right)^{-2}$
 $= \left(\frac{1}{z^6}\right)^{-2}$
 $\frac{1^2}{(z^6)^{-2}} = \frac{1}{z^{-12}}$
 $= z^{12}$

Day 1: Properties of Exponents

1) An _____ is a quantity that shows the number of times a given number is being _____ by _____.

Examples: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$

$3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z = 3^3 x^4 y^2 z$

2) An exponential term contains a base and a superscript power (also known as an exponent).

Example: a^n : a is the base, n is the power

x^{17} : x is the base, 17 is the power.

x^3 - superscript exponent.
 a^2 - subscript name

Let's put it all together!

Simplify. Your answer should contain only positive exponents.

3) $(n^{-2} \cdot n)^{-4}$
 $(n^{-1})^{-4}$
 $n^{-1 \cdot -4}$
 n^4

4) $(m^2 m^7)^2$
 $(m^9)^2$
 m^{18}

5) $a^2 \cdot (a^2)^2$
 $a^2 \cdot a^4$
 a^6

6) $n^{-2} n^2$
 $n^0 = 1$

7) $x^{-3} \cdot (x^2)^0$
 $x^{-3} \cdot 1$
 $\frac{1 \cdot x^{-3}}{1} = \frac{1}{x^3}$
 $\frac{x^{-3}}{1} \cdot \frac{x^3}{x^3} = \frac{x^0}{x^3}$
 $= \frac{1}{x^3}$

8) $(n^{-2})^{-2} \cdot n^{-5}$
 $(n^{-2 \cdot -2}) \cdot n^{-5}$
 $n^4 \cdot n^{-5} = n^{-1}$
 $= \frac{1}{n^1}$

$$9) \frac{b^2}{(b^2)^2 \cdot b^2} = \frac{b^2}{b^4 \cdot b^2} = \frac{b^2}{b^6}$$

$$= \frac{\cancel{b^2}}{b^4 \cdot \cancel{b^2}} = \frac{1}{b^4} = \frac{1}{b^4 \cdot b^{-2}} = \frac{1}{b^2}$$

$$10) \frac{(m^2 m^{-1})^{-1}}{(m^2)^0} = \left(\frac{m^1}{1}\right)^{-1} = \frac{m^{-1}}{1}$$

$$= \frac{1}{m}$$

$$11) \frac{(b^2)^2}{bb^{-2}b^{-1}} = \frac{b^4}{b^1 b^{-2} b^{-1}}$$

$$= \frac{b^4}{b^{-2}} = b^{4+2} = b^6$$

$$12) \frac{(2k^2)^2}{k^{-2} \cdot k} = \frac{2^2 \cdot k^{2 \cdot 2}}{k^{-1}}$$

$$= \frac{2^2 \cdot k^4}{k^{-1}} = 2^2 \cdot k^4 \cdot k^1 = 4k^5$$

$$13) \frac{(aa^0)^{-2}}{a} = \frac{(a^1 a^0)^{-2}}{a}$$

$$= \frac{(a^1)^{-2}}{a} = \frac{a^{-2}}{a^1} = \frac{1}{a^1 a^2} = \frac{1}{a^3}$$

$$14) \frac{b^0 \cdot (b^{-2})^2}{b^{-2}} = \frac{1 \cdot b^{-4}}{b^{-2}}$$

$$= \frac{1}{b^{-2} b^4} = \frac{1}{b^2}$$

$$15) \left(\frac{x^6}{x^{-2} x^5}\right)^{-2}$$

$$\left(\frac{x^6}{x^3}\right)^{-2}$$

$$(x^{6-3})^{-2}$$

$$(x^3)^{-2} = x^{-6} = \frac{1}{x^6}$$

$$16) \left(\frac{m}{m^{-1} m^2}\right)^3 = \left(\frac{m}{m}\right)^3$$

$$= (1)^3 = 1$$