

Day 5: Imaginary and Complex Numbers

Date _____

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What is an imaginary number?

- 1) An imaginary number is created when a negative is in the square root

Check out this great website as a resource:

<http://www.mathsisfun.com/numbers/imaginary-numbers.html>

So this means that $i = \sqrt{-1}$.

- 2) Imaginary numbers follow a pattern with their exponents:

$$\begin{aligned} i &= i = \sqrt{-1} \\ -1 &= i^2 = (\sqrt{-1})^2 = -1 \\ -i &= i^3 = i \cdot i^2 = \sqrt{-1} \cdot -1 = -\sqrt{-1} \\ 1 &= i^4 = i^2 \cdot i^2 = -1 \cdot -1 \end{aligned}$$

- 3) Then it repeats:

$$\begin{aligned} i^5 &= i \\ i^6 &= -1 \\ i^7 &= -i \\ i^8 &= 1 \end{aligned}$$

- 4) After what power do the imaginary numbers start to repeat?

Simplify the following.

5) $i^0 = 1$

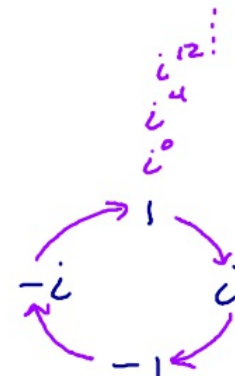
6) $i^{57} = i$

7) $i^{34} = -1$

8) $i^{85} = i$

9) $i^{47} = -i$

10) $i^{12} = 1$



Rewrite the following as imaginary.

11) $\sqrt{-25} = \sqrt{-1} \cdot \sqrt{25}$
 $i \cdot 5 = 5i$

12) $\sqrt{-81} = \sqrt{-1} \cdot \sqrt{81}$
 $= 9i$

13) $\sqrt{-121}$

14) $\sqrt{-9}$

$$15) \sqrt{-32} = \sqrt{-1} \cdot \sqrt{32} = i \cdot \sqrt{16 \cdot 2} = 4i\sqrt{2}$$

$$16) \sqrt{-45} = \sqrt{-1} \cdot \sqrt{45} = i \cdot \sqrt{9 \cdot 5} = 3i\sqrt{5}$$

$$17) \sqrt{-11} = i\sqrt{11}$$

$$18) \sqrt{-23} = i\sqrt{23}$$

What is a Complex Number?

19) A number with both a real part and an imaginary part written in the form of $a+bi$.

Identify the real part and the imaginary part of the following.

$$20) \frac{6+5i}{R \quad im} \quad R=6 \quad im=5i$$

$$21) \frac{8-3i}{R \quad im} \quad R=8 \quad im=-3i$$

$$22) \frac{-4-7i}{R \quad im} \quad R=-4 \quad im=-7i$$

$$23) \frac{-1+13i}{R \quad im} \quad R=-1 \quad im=13i$$

Adding and Subtracting Complex Numbers

24) We add and subtract Complex Numbers just like we do Polynomials, which we did in Day 3 Notes.

Add or Subtract the following Complex Numbers.

$$25) (-3-9i) + (11-7i)$$

$$\boxed{8-16i}$$

$$26) (4+i) + (-6+2i)$$

$$\boxed{-2+3i}$$

$$27) (-1-3i) + (3-6i)$$

$$\boxed{2-9i}$$

$$28) (1+2i) - (-5-12i)$$

$$\boxed{6+14i}$$

$$29) (-11+4i) - (-10+i)$$

$$-11+4i+10-i$$

$$\boxed{-1+3i}$$

$$30) (6-11i) - (11-6i)$$

Multiplying Complex Numbers

31) We multiply Complex Numbers like we multiply Polynomials.

The only difference will be the term with the i^2 .

REMEMBER $i^2 = -1$.

Multiply the following Complex Numbers.

32) $(-7 + 2i)(-7 - 4i)$

$$49 + 28i - 14i - 8i^2$$

$$49 + 14i + 8(1)$$

$$\boxed{57 + 14i}$$

a bi

33) $(6 + 8i)(-6 - 2i)$

$$-36 - 12i - 48i - 16i^2$$

$$-36 - 60i + 16(1)$$

$$\boxed{-20 - 60i}$$

34) $(-6 + 5i)^2 = (-6 + 5i)(-6 + 5i)$

$$36 - 30i - 30i + 25i^2$$

$$36 - 60i + 25(-1)$$

$$\boxed{11 - 60i}$$

a bi

35) $(4 - i)(1 - 3i)$

$$4 - 12i - i + 3i^2$$

$$4 - 13i - 3$$

$$\boxed{1 - 13i}$$

36) $3(-8i)(2 - 7i)$

$$-24i(2 - 7i)$$

$$-48i + 168i^2$$

$$-168 - 48i$$

37) $(i)(-6i)(7 - 2i)$

$$-6i^2(7 - 2i)$$

$$-6(-1)(7 - 2i)$$

$$6(7 - 2i)$$

$$\boxed{42 - 12i}$$

38) $(-7 - 8i)(3 - 6i) - 7(8i)(6 - 7i)$

39) $(-1 + 2i) + (6 - i) + (4 + 4i)(8 + i)$

$$-1 + 2i + 6 - i + 32 + 4i + 32i + 4i^2$$

$$\boxed{33 + 37i}$$

