Day 5: Imaginary and Complex Numbers

Date________________

What is an imaginary number?

1) An imaginary number is created when a negative is in the square root

Check out this great website as a resource:
http://www.mathsisfun.com/numbers/imaginary-numbers.html

So this means that $i = \sqrt{-1}$.

2) Imaginary numbers follow a pattern with their exponents:

$i = i = \sqrt{-1}$

$-1 = i^2 = (\sqrt{-1})^2 = -1$

$-i = i^3 = i \cdot i^2 = \sqrt{-1} \cdot -1 = -\sqrt{-1}$

$1 = i^4 = i^2 \cdot i^2 = -1 \cdot -1$

3) Then it repeats:

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

4) After what power do the imaginary numbers start to repeat?

i^{12} ⋮

i^4

i^0

1

$-i$ i

-1

Simplify the following.

5) $i^0 = 1$

6) $i^{57} = i$

7) $i^{34} = -1$

8) $i^{85} = i$

9) $i^{47} = -i$

10) $i^{12} = 1$

Rewrite the following as imaginary.

11) $\sqrt{-25} = \sqrt{-1} \cdot \sqrt{25}$
$i \cdot 5 = 5i$

12) $\sqrt{-81} = \sqrt{-1} \cdot \sqrt{81}$
$= 9i$

13) $\sqrt{-121}$

14) $\sqrt{-9}$

15) $\sqrt{-32}$ $= \sqrt{-1} \cdot \sqrt{32}$ $= 4\sqrt{2}$
$4i\sqrt{2}$ i 16 2
44

16) $\sqrt{-45}$ $= \sqrt{-1} \cdot \sqrt{45}$ $3\sqrt{5}$
$3i\sqrt{5}$ 9 5
3 3

17) $\sqrt{-11}$ $= i\sqrt{11}$

18) $\sqrt{-23}$ $= i\sqrt{23}$

What is a Complex Number?

19) A number with both a real part and an imaginary part written in the form of $a + bi$.

Identify the real part and the imaginary part of the following.

20) $\underset{R}{\underline{6}} + \underset{im}{\underline{5i}}$ $R = 6$ $im = 5i$

21) $\underline{8} - \underline{3i}$ R 8 im $-3i$

22) $-4 - 7i$ $R = -4$ $im = -7i$

23) $-1 + 13i$ $R = -1$ $im = 13i$

Adding and Subtracting Complex Numbers

24) We add and subtract Complex Numbers just like we do Polynomials, which we did in Day 3 Notes.

Add or Subtract the following Complex Numbers.

25) $(-3 - 9i) + (11 - 7i)$
$\boxed{8 - 16i}$

26) $(4 + i) + (-6 + 2i)$
$\boxed{-2 + 3i}$

27) $(-1 - 3i) + (3 - 6i)$
$\boxed{2 - 9i}$

28) $(1 + 2i) - (-5 - 12i)$
$1 + 2i + 5 + 12i$
$\boxed{6 + 14i}$

29) $(-11 + 4i) - (-10 + i)$
$-11 + 4i + 10 - i$
$\boxed{-1 + 3i}$

30) $(6 - 11i) - (11 - 6i)$

Multiplying Complex Numbers

31) We multiply Complex Numbers like we multiply Polynomials.

The only difference will be the term with the i^2. REMEMBER $i^2 = -1$.

Multilpy the following Complex Numbers.

32) $(-7 + 2i)(-7 - 4i)$

$49 + 28i - 14i - 8i^2 = -1$

$49 + 14i + 8(+1)$

$\boxed{57 + 14i}$

a bi

33) $(6 + 8i)(-6 - 2i)$

$-36 - 12i - 48i - 16i^2$ -1

$-36 - 60i + 16(+1)$

$\boxed{-20 - 60i}$

34) $(-6 + 5i)^2$ $= (-6+5i)(-6+5i)$

$36 - 30i - 30i + 25i^2$

$36 - 60i + 25(-1)$

$\boxed{11 - 60i}$

a bi

35) $(4 - i)(1 - 3i)$

$4 - 12i - i + 3i^2$

$4 - 13i - 3$

$\boxed{1 - 13i}$

36) $3(-8i)(2 - 7i)$

$-24i(2-7i)$

$-48i + 168i^2$

$-168 - 48i$

37) $(i)(-6i)(7 - 2i)$

$-6i^2(7-2i)$

$-6(-1)(7-2i)$

$6(7-2i)$

$\boxed{42 - 12i}$

38) $(-7 - 8i)(3 - 6i) - 7(8i)(6 - 7i)$

39) $(-1 + 2i) + (6 - i) + (4 + 4i)(8 + i)$

$-1 + 2i + 6 - i + 32 + 4i + 32i + 4i^2$

-4

$\boxed{33 + 37i}$

