

Day 1: Factoring Quadratics

1) Factoring is taking a quadratic expression and writing it in the form $(x+m)(x+n)$.

It is often referred to as undistributing.

Notice that in our quadratic expression:

The middle term is always $mx + nx$.

The constant term is always $m * n$.

Standard Form.
 $ax^2 + bx + c$

Factored Form

$a(x+m)(x+n)$

$x^2 + \underline{nx} + \underline{mx} + mn$

Factor each completely.

2) $p^2 + 11p + 28$
 $(p+4)(p+7)$

<u>28</u>	
1	• 28
2	• 14
4	• 7

3) $x^2 - 8x + 16$
 $(x-4)(x-4)$

<u>16</u>	
± 1	• ± 16
± 2	• ± 8
± 4	• ± 4

4) $a^2 - 7a - 28$
 $(a+) (a-)$

<u>28</u>	
± 1	• ± 28
± 2	• ± 14
± 4	• ± 7

Not Factorable

5) $v^2 - 1v - 20$
 $(v+4)(v-5)$
 $(v-5)(v+4)$

<u>20</u>	
1	• 20
2	• 10
4	• 5

6) $x^2 + 8x + 12$
 $(x+2)(x+6)$

<u>12</u>	
1	• 12
2	• 6
3	• 4

7) $r^2 + 7r + 6$
 $(r+1)(r+6)$

<u>6</u>	
1	• 6
2	• 3

8) $n^2 + 15n + 56$
 $(n+7)(n+8)$

9) $v^2 + 2v - 20$
 $(v+) (v-)$

<u>20</u>	
1	• 20
2	• 10
4	• 5

Not Factorable.

10) $a^2 - 3a + 0$
 $(a+0)(a-3)$
 $a(a-3)$

11) $r^2 - 9r$
 $r(r-9)$

12) This process only works if the Leading Coefficient is 1. When we get a leading coefficient that is not 1 we must get our leading coefficient to be 1.

For example in the polynomial $\frac{5x^2}{5} + \frac{25x}{5} + \frac{30}{5}$ the leading coefficient is a 5, so can we divide 5 from all of the terms?

$$5(x^2 + 5x + 6) \quad \begin{array}{r} 6 \\ 1 \cdot 6 \\ 2 \cdot 3 \end{array}$$

$$5(x+2)(x+3)$$
~~$$5(x+1)(x+6)$$~~

In the resulting quadratic expression all three terms are divisible by 5, so we need to "undistribute" that 5 before we factor the remaining expression.

Factor each completely.

13) $\frac{5m^2}{5} + \frac{75m}{5} + \frac{280}{5}$

$$5(m^2 + 15m + 56)$$

$$5(m+7)(m+8)$$

$$\begin{array}{r} 56 \\ 1 \cdot 56 \\ 2 \cdot 28 \\ 4 \cdot 14 \\ 7 \cdot 8 \end{array}$$

14) $\frac{4x^2}{4} - \frac{8x}{4} - \frac{32}{4}$

$$4(x^2 - 2x - 8)$$

$$4(x+2)(x-4)$$

$$\begin{array}{r} 8 \\ 1 \cdot 8 \\ 2 \cdot 4 \end{array}$$

15) $\frac{-6r^2}{-6} - \frac{42r}{-6} + \frac{48}{-6}$

$$-6(r^2 + 7r - 8)$$

$$-6(r+8)(r-1)$$

$$\begin{array}{r} 8 \\ 1 \cdot 8 \\ 2 \cdot 4 \end{array}$$

16) $\frac{-3x^2}{-3} - \frac{18x}{-3} + \frac{120}{-3}$

$$-3(x^2 + 6x - 40)$$

$$-3(x+10)(x-4)$$

$$\begin{array}{r} 40 \\ 1 \cdot 40 \\ 2 \cdot 20 \\ 4 \cdot 10 \\ 5 \cdot 8 \end{array}$$

17) $\frac{-2r^2}{-2} + \frac{10r}{-2} + \frac{72}{-2}$

$$-2(r^2 - 5r - 36)$$

$$-2(r+4)(r-9)$$

$$\begin{array}{r} 36 \\ 1 \cdot 36 \\ 2 \cdot 18 \\ 3 \cdot 12 \\ 4 \cdot 9 \\ 6 \cdot 6 \end{array}$$

18) $\frac{-x^2}{-1} + \frac{9x}{-1} - \frac{20}{-1}$

$$-(x^2 - 9x + 20)$$

$$-(x-4)(x-5)$$

$$\begin{array}{r} 20 \\ 1 \cdot 20 \\ 2 \cdot 10 \\ 4 \cdot 5 \end{array}$$

19) $2v^2 + 12v + 18$

20) $3x^2 - 30x + 75$

Special Cases

21) Multiply $(x+2)(x-2)$
 $x^2 - 2x + 2x - 4$
 $x^2 - 4$ $x^2 + 0x - 4$

Multiply $(x+10)(x-10)$
 $x^2 - 10x + 10x - 100$
 $x^2 - 100$ $x^2 + 0x - 100$

What patterns do you notice?

Difference of Squares

22) If you have to factor any quadratic in the form $a^2 - b^2$ it will always factor to be $(a+b)(a-b)$.

Factor each completely.

23) $k^2 - 1$

$$(k+1)(k-1)$$
$$(k-1)(k+1)$$

24) $n^2 - 16$

$$(n+4)(n-4)$$

25) $a^2 - 25$

$$(a+5)(a-5)$$

26) $v^2 - 4$

$$(v-2)(v+2)$$

27) $\frac{45a^2 - 20}{5 \cdot 5}$

$$5(9a^2 - 4)$$
$$5(3a+2)(3a-2)$$

28) $16x^2 - 100$

$$(4x+10)(4x-10)$$

29) $\frac{12n^2 - 27}{3 \cdot 3}$

$$3(n^2 - 9)$$
$$3(n+3)(n-3)$$

30) $4r^2 - 25$

$$(2r+5)(2r-5)$$

31) $49r^2 - 25$

$$(7r+5)(7r-5)$$

32) $121b^2 - 4$

$$(11b+2)(11b-2)$$