

Conditional Probability

Use the table to the right about ear piercings:

Gender	Pierced Ears?		Total
	Yes	No	
Male	19	71	90
Female	84	4	88
Total	103	75	178

1. If we know that a randomly selected student has pierced ears, what is the probability that the student is male?

a. First how many students have pierced ears?
103

b. We restrict our interest to just this group. And then, how many males are there with pierced ears? 19

c. So we can say: $P(\text{male} \mid \text{has pierced ears}) = \frac{19}{103}$

2. If we know that a randomly selected student is male, what's the probability that the student has pierced ears?

a. First how many students are male?
90

b. We restrict our interest to just this group. And then, how many of the males have pierced ears? 19

c. So we can say: $P(\text{has pierced ears} \mid \text{is male}) = \frac{19}{90}$

These two questions sound alike, but they are asking about two very different things.

The probability that one event happens given that another event is already known to have happened is called a Conditional Probabilities.

Suppose we know that event A has happened. Then the probability that event B happens given that event A has happened is denoted by $P(B|A)$.

To find $P(B|A) = \frac{\text{Number of outcomes in (A and B)}}{\text{Number of outcomes in A}}$

To find $P(A|B) = \frac{\text{Number of outcomes in (A and B)}}{\text{Number of outcomes in B}}$

So using this notation if $P(M)=P(\text{male})$ and $P(P)=P(\text{pierced ears})$:

$P(\text{male} \mid \text{has pierced ears}) = P(M|P) = \frac{19}{103}$

$P(\text{has pierced ears} \mid \text{is male}) = P(P|M) = \frac{19}{90}$

A random group of 50 Australian high school students was selected to complete a survey. The two-way table displays data on the gender and dominant hand of each student.

Gender	Dominant Hand		Total
	Right (R)	Left (L)	
Male (M)	20	3	23
Female (F)	23	4	27
Total	43	7	50

$$3. P(M) = \frac{23}{50} = 46\%$$

$$4. P(M|L) = \frac{3}{7} = 43\%$$

$$5. P(M \cap L) = \frac{3}{50} = 6\%$$

$$6. P(L) = \frac{7}{50} = 14\%$$

$$7. P(L|M) = \frac{3}{23} = 13\%$$

$$8. P(M \cup R) = \frac{23 + 43 - 20}{50} = \frac{46}{50} = 92\%$$

$$9. P(F) =$$

$$10. P(F \cap R) =$$

$$11. P(R|F) =$$

$$12. P(F|L) =$$

13. Which of the previous probabilities is a **joint probability**? $P(\cup)$ or $P(\cap)$
5, 8, 10

14. Which of the previous probabilities is a **marginal probability**? $P(\cup)$
3, 6, 9

15. Which of the previous probabilities is a **conditional probability**? $P(|)$
4, 7, 11, 12

Students at the University of New Harmony received 10,000 course grades last semester. The two-way table below breaks down these grades by which school of the university taught the course.

School that teaches the course:	Grade Level			TOTAL
	Grade A (A)	Grade B (B)	Grade is Below a B	
Liberal Arts (L)	2,142	1,890	2,268	6300
Engineering & Physical Science (EPS)	368	432	800	1600
Health and Human Services (H)	882	630	588	2100
TOTAL	3392	2952	3656	10000

12. $P(\text{Below a B}) = \frac{2952}{10000} = 29.52\%$

13. $P(\text{EPS}) = \frac{1600}{10000} = 16\%$

14. $P(\text{EPS}|\text{Below a B}) = \frac{432}{2952} = 14.6\%$

15. $P(\text{Below a B}|\text{EPS}) = \frac{432}{1600} = 27\%$

16. Which of the probabilities tells us whether this college's EPS students tend to earn grades that are below a B?

The RMS Titanic had ~~221~~²¹¹⁵ people on board ship (including the children), yet there was lifeboat space for only 1178. Various factors on that fateful day also made it so many lifeboats were not full when they were launched. (Data from http://en.wikipedia.org/wiki/RMS_Titanic Art by Willy Stower)

Gender	Survival		Total
	Survived (S)	Lost (L)	
Male (M)	338	1352	1690
Female (F)	316	109	425
Total	654	1461	2115



20. $P(M) = \frac{1690}{2115} = 80\%$

21. $P(S) = \frac{654}{2115} = 31\%$

22. $P(S|M) = \frac{338}{1690} = 20\%$

23. $P(F) = \frac{425}{2115} = 20\%$

24. $P(S|F) = \frac{316}{425} = 74\%$

25. $P(F|S) = \frac{316}{654} = 48\%$

26. Historically when populations are in danger, societies protect women. Does the Titanic data support this priority? Explain why or why not.