

Solving from Vertex Form and the Fundamental Theorem of Algebra

How many complex solutions does $x + 1 = 5$ have? Find them.

1 solution

$$\begin{array}{r} -1 \quad -1 \\ x = 4 \end{array}$$

How many complex solutions does $x^2 - 4 = 0$ have? Find them.

2 solutions

$$\sqrt{x^2 = 4}$$

$$x = \pm 2$$

$$\begin{array}{r} +4 \quad +4 \\ x^2 = 4 \end{array}$$

$$(x-2)(x+2) = 0$$

$$x = 2 \quad x = -2$$

$a + bi$

Complex ts		
\mathbb{R}	$\mathbb{I}m$	
Q	I	i
W	$\sqrt{2}$	$\sqrt{-1}$
N		

Corollary to the Fundamental Theorem of Algebra:

A Polynomial of Nth degree has N complex solutions

What is the degree of each equation below?

How many complex solutions should each equation below have?

$$x^2 + 3x - 4 = 0$$

\rightarrow 2 solutions

$$x^5 - 3x^2 + 5 = 0$$

\rightarrow 5 solutions

For each equation below do the following: A) identify the form the equation is in B) Solve the equation from vertex form C) identify each equation's solution(s) as either real (R) or imaginary (i) AND as either rational (Q) or irrational (I)

Standard Vertex

$$x^2 + 0x - 25 = 0 \quad (x-0)^2 - 25 = 0$$

1. $x^2 - 25 = 0$

A. Std. or Vertex

B.
$$\begin{array}{r} x^2 - 25 = 0 \\ +25 \quad +25 \\ \hline \sqrt{x^2} = \sqrt{25} \\ x = \pm 5 \end{array}$$

C. \mathbb{R}, Q

3. $x^2 + 9 = 0$

A. Std or Vertex

B.
$$\begin{array}{r} x^2 + 9 = 0 \\ -9 \quad -9 \\ \hline \sqrt{x^2} = \sqrt{-9} = \sqrt{-1} \cdot \sqrt{9} \\ x = \pm 3i \end{array}$$

$i \cdot 3$

C. $\mathbb{I}m, Q$

2. $2x^2 - 6 = 0$

A. Std. Vertex

B.
$$\begin{array}{r} 2x^2 - 6 = 0 \\ +6 \quad +6 \\ \hline 2x^2 = 6 \\ \frac{2x^2}{2} = \frac{6}{2} \\ \sqrt{x^2} = \sqrt{3} \\ x = \pm\sqrt{3} \end{array}$$

C. \mathbb{R}, \mathbb{I}

4. $(x-1)^2 + 4 = 6$

A. Vertex

B.
$$\begin{array}{r} (x-1)^2 + 4 = 6 \\ -4 \quad -4 \\ \hline \sqrt{(x-1)^2} = \sqrt{2} \\ x-1 = \pm\sqrt{2} \\ +1 \quad +1 \\ \hline x = 1 \pm\sqrt{2} \end{array}$$

C. \mathbb{R}, \mathbb{I}

$$\begin{array}{r} (x-1)(x-1) = 2 \\ x^2 - 1x - 1x + 1 = 2 \\ -1 \quad -1 \\ \hline x^2 - 1x - 1x = 1 \\ \sqrt{x^2} - 2x = \frac{1}{2} \\ \frac{x^2}{2} + x = \frac{1}{2} \end{array}$$

?

5. $(x-4)^2 - 1 = 0$

A. Vertex

B.

$$a^2 - 1 = 0$$

$$\frac{+1 \quad +1}{\sqrt{a^2} = 1}$$

$$a = \pm 1$$

$$(x-4) = \frac{\pm 1}{+4 \quad +4}$$

$$x = 4 \pm 1 \begin{matrix} \leftarrow 4+1 \\ \leftarrow 4-1 \end{matrix}$$

$$x = 5, 3$$

c. \mathbb{R}, \mathbb{Q}

7. $9x^2 + 40 = -24$ Std or Vertex

A. $9x^2 + 64 = 0$

B.

$$\sqrt{x^2} = \sqrt{\frac{-64}{9}} = \sqrt{-\frac{64}{9}}$$

$$x = \pm \frac{8}{3}i$$

c. \mathbb{Im}, \mathbb{Q}

9. $x^2 - 6x + 9 = -9$ Std.

A. $x^2 - 6x + 18 = 0$

B.

	$x-3$	
x	x^2	$-3x$
-3	$-3x$	9
		$+9$

$$(x-3)^2 + 9 = 0$$

$$\frac{-9 \quad -9}{\sqrt{(x-3)^2} = -9}$$

$$x-3 = \pm 3i$$

$$\frac{+3 \quad +3}{x = 3 \pm 3i}$$

c. \mathbb{Im}, \mathbb{Q}

Vertex

6. $5(x-1)^2 - 3 = 0$

A. $5(x-1)^2 = 3$

B.

$$\sqrt{(x-1)^2} = \sqrt{\frac{3}{5}}$$

$$(x-1) = \pm \frac{\sqrt{3}}{\sqrt{5}}$$

$$x = 1 \pm \sqrt{\frac{3}{5}}$$

c. \mathbb{R}, \mathbb{IR}

8. $3x^2 + 12x + 6 = 0$ - Standard

A. $3(x^2 + 4x + 2) = 0$

B.

	$x+2$	
x	x^2	$2x$
$+2$	$2x$	4
		-2

$$3[(x+2)^2 - 2] = 0$$

$$3(x+2)^2 - 6 = 0$$

$$\frac{+6 \quad +6}{3(x+2)^2 = 6}$$

$$\sqrt{(x+2)^2} = \sqrt{2}$$

$$x+2 = \pm \sqrt{2}$$

$$\frac{-2 \quad -2}{x = -2 \pm \sqrt{2}}$$

c. \mathbb{R}, \mathbb{IR}

10. $-x^2 + 4x - 7 = 0$ Std.

A. $-1(x^2 - 4x + 7) = 0$

B.

	$x-2$	
x	x^2	$-2x$
-2	$-2x$	4
		$+3$

$$-(x-2)^2 + 3 = 0$$

$$-(x-2)^2 - 3 = 0$$

$$\frac{-3 \quad -3}{\sqrt{(x-2)^2} = -3}$$

$$x-2 = \pm i\sqrt{3}$$

$$\frac{+2 \quad +2}{x = 2 \pm i\sqrt{3}}$$

c. \mathbb{Im}, \mathbb{IR}