

## Solving using Quadratic Formula - NO CALCULATOR!

- 1) Given the standard form of a quadratic equation,  $(ax^2 + bx + c = 0)$ , derive the quadratic formula by completing the square.

$$ax^2 + bx + c = 0$$

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a} = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$\frac{\frac{b}{a}}{2} = \frac{b}{2a}$$

$x$	$x + \frac{b}{2a}$	
	$x^2$	$\frac{b}{2a}x$
$+\frac{b}{2a}$	$\frac{b}{2a}x$	$\frac{b^2}{4a^2}$

$-\frac{b^2+4ac}{4a^2}$

$$\frac{b^2}{4a^2} + \quad = \frac{c}{a}$$

$$\frac{b^2}{4a^2} + \frac{b^2+4ac}{4a^2} = \frac{4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{-b^2+4ac}{4a^2} = 0$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$-\frac{b}{2a} \quad -\frac{b}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

- 2) There is one more method we can use to solve quadratic equations. It is used when the equation is in Standard form, meaning  $ax^2 + bx + c = 0$ . It's called the quadratic formula, and you need to memorize it!

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 3) A part of the quadratic equation that just includes what is under the radical denominator  $b^2 - 4ac$  is called the discriminant. And can tell us about our solutions.

If the discriminant is a positive perfect square our solutions will be rational and real.

$$\sqrt{16} = \pm 4$$

If the discriminant is a positive non-perfect square our solutions will be Irrat. & real.

$$\sqrt{17}$$

If the discriminant is a negative perfect square our solutions will be rational & Imag.

$$\sqrt{-25} = 5i$$

If the discriminant is a negative non-perfect square our solutions will be Irrat. & Imag

$$\sqrt{-37}$$

- 4) Use this formula to solve the following equation:

$$x^2 + 4x - 9 = 0$$

Identify the following:

$$a = 1 \quad b = 4 \quad c = -9$$

Now plug it in:

$$x = \frac{-4 \pm \sqrt{4^2 + 4(1)(-9)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 + 36}}{2}$$

$$= \frac{-4 \pm \sqrt{52}}{2} = \frac{-4 \pm 2\sqrt{13}}{2} = \frac{-4}{2} \pm \frac{2\sqrt{13}}{2}$$

And solve for x:

$$x: -2 + \sqrt{13}, -2 - \sqrt{13}$$

$$= -2 \pm \sqrt{13}$$

$$\begin{array}{l} \sqrt{52} \\ \swarrow \quad \searrow \\ 2\sqrt{13} \end{array} \quad \begin{array}{l} 4+6 \\ \leftarrow 2 \rightarrow \\ \frac{4}{2} + \frac{6}{2} \end{array}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve each equation from standard form. Then classify each equation's solution(s) as either rational real (QR), irrational real (IR), rational imaginary (Qi) or irrational imaginary (Ii).

5)  $2k^2 - k - 1 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1+8}}{4}$$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1 \pm 3}{4} \quad \begin{cases} \frac{1+3}{4} = 1 \\ \frac{1-3}{4} = -\frac{1}{2} \end{cases}$$

$$x = -\frac{1}{2}, 1$$

7)  $2p^2 = -10$

$$\frac{+10+10}{2p^2+10=0}$$

$$2p^2 + 0p + 10 = 0$$

$$\frac{-0 \pm \sqrt{0^2 - 4(2)(10)}}{2(2)}$$

$$\frac{\pm \sqrt{-80}}{4} = \pm \frac{4i\sqrt{5}}{4} = \pm i\sqrt{5}$$

$$\sqrt{-80} = i\sqrt{2 \cdot 40} = i\sqrt{2 \cdot 2 \cdot 20} = i\sqrt{2 \cdot 2 \cdot 2 \cdot 10} = 2i\sqrt{5}$$

Calc: Error

6)  $m^2 - 2m - 3 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+12}}{2}$$

$$= \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2} \quad \begin{cases} \frac{2+4}{2} = \frac{6}{2} = 3 \\ \frac{2-4}{2} = \frac{-2}{2} = -1 \end{cases}$$

$$x = 1, 3$$

8)  $n^2 = 10n - 7$

$$n^2 - 10n + 7 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100-28}}{2}$$

$$= \frac{10 \pm \sqrt{72}}{2}$$

$$= \frac{10 \pm 6\sqrt{2}}{2} = 5 \pm 3\sqrt{2}$$

$$\sqrt{72} = \sqrt{2 \cdot 36} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$$

9)  $2n^2 = -7n + 60$

$$2n^2 + 7n - 60 = 0$$

$$\frac{-7 \pm \sqrt{(-7)^2 + 4(2)(+60)}}{2(2)}$$

$$\frac{-7 \pm \sqrt{49 + 8(60)}}{4}$$

$$\frac{-7 \pm \sqrt{49 + 480}}{4}$$

$$\frac{-7 \pm \sqrt{529}}{4} \quad \begin{cases} \frac{-7+23}{4} = \frac{16}{4} = 4 \\ \frac{-7-23}{4} = \frac{-30}{4} = -\frac{15}{4} \end{cases}$$

10)  $7n^2 + 3 = -3n$

$$7n^2 + 3n + 3 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(7)(3)}}{2(7)}$$

$$x = \frac{-3 \pm \sqrt{9-84}}{14}$$

$$= \frac{-3 \pm \sqrt{-75}}{14} = \frac{-3 \pm 5i\sqrt{3}}{14}$$

11) Go back and find the discriminant for #5 \_\_\_\_\_, #7 \_\_\_\_\_,

#8 \_\_\_\_\_, and #10 \_\_\_\_\_.