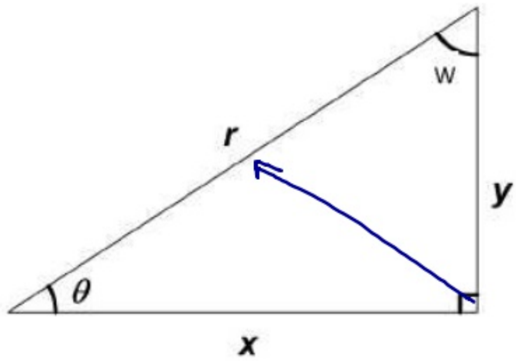
Secondary 2 Honors - Trigonometry Unit Day 3 In Class Notes

Creating Trig Identities

To the right is a typical right triangle. Use the variables provided to find the following trigonometric ratios.



$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

Identity: A rule that is always true, no matter the input. More true than an equation, which is only true for certain inputs.

Create an identity between tangent and sine and cosine: (hint: don't worry about the identity part right now, just try and create an equation that shows how the 3 trig functions compare)

Tangent Identity: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \frac{y}{x}$$

<u>Proving/Finding the Pythagorean Identity</u>

1. Apply the Pythagorean Theorem to the triangle and create a true statement.

$$a^2 + b^2 = c^2 \implies \frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

2. Divide both sides of the equation by r^2.

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

3. Simplify the equation so that there are only two exponents.

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

4. Rewrite the equation only using Θ. (*x*, *y*, and *r* cannot appear in the equation.)

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

Pythagorean Identity:

$$\cos^2\theta + \sin^2\theta = 1$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cos^2\theta + \sin^2\theta = 1$

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$\tan^{-1}\theta = \frac{\cos\theta}{\sin\theta}$

Use the tangent and Pythagorean identities to simplify the following expressions as much possible.

1. $\cos^2\theta\tan^2\theta + \cos^2\theta$

$\frac{\cancel{\cos^2\theta}}{1}\left(\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\right) + \cos^2\theta$

$\sin^2\theta + \cos^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

1

2. $\cos^2\theta - 1$

$\cos^2\theta - (\cos^2\theta + \sin^2\theta)$

$\cos^2\theta - \cos^2\theta - \sin^2\theta$

$-\sin^2\theta$

3. $6(\cos^2\theta + \sin^2\theta) - 4$

$6(1) - 4$

$6 - 4$

2

4. $\cos\theta\sin^2\theta - \cos\theta$

Think

$\frac{2}{2}x - \frac{2}{3}$

$2(x-1)$

$\cos\theta(\sin^2\theta - 1)$

$\cos\theta(\sin^2\theta - (\cos^2\theta + \sin^2\theta))$

$\cos\theta(\cancel{\sin^2\theta} - \cos^2\theta - \cancel{\sin^2\theta})$

$\cos\theta(-\cos^2\theta)$

$-\cos^3\theta$

5. $\frac{\sin\theta\cos\theta}{1-\cos^2\theta}$

$\frac{\sin\theta\cos\theta}{\cancel{\cos^2\theta} + \sin^2\theta - \cancel{\cos^2\theta}}$

$\frac{\cancel{\sin\theta}\cos\theta}{\sin^2\theta}$

Think!

$\frac{xy}{x^2} = \frac{\cancel{x}y}{\cancel{x}x} = \frac{y}{x}$

$\frac{\cos\theta}{\sin\theta} = \tan^{-1}\theta$

6. $-\sin\theta\cos\theta\tan\theta - \cos^2\theta$

$-\sin\theta\,\cancel{\cos\theta}\frac{\sin\theta}{\cancel{\cos\theta}} - \cos^2\theta$

$-\frac{\sin^2\theta}{-1} - \frac{\cos^2\theta}{-1}$

$-(\sin^2\theta + \cos^2\theta)$

$-(1)$

-1